Effects of a transverse magnetic field on macroscopic quantum tunneling.

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ABSTRACT

We present a comprehensive study of the effect of a transverse magnetic field on the macroscopic quantum tunneling of two identical coupled particles with dissipation. Two coupled particles of identical masses but opposite charges in the plane, in the presence of a constant transverse external magnetic field and an external potential interacting with a bath of harmonic oscillators are studied. In this case, the problem cannot be mapped to a one-dimensional problem, it strictly remains two-dimensional. We calculate the exact effective action both for the case of linear coupling to the bath and without a linear coupling to the bath using imaginary time path integral and Leggett's prescription at $T = 0$. In the limit of zero magnetic field we recover a two dimensional version of the result found in [3] for the case of two identical particles. We find that in the limit of weak and strong dissipation, the effective action reduces to a two dimensional version of the Caldeira-Leggett form in terms of the reduced mass and magnetic field. The case of Ohmic dissipation with the motion of the two particles damped by the Ohmic frictional constant η is studied in detail.

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INTRODUCTION

Macroscopic quantum tunneling with dissipation has become the subject of interest in quantum statistical mechanics and condensed matter physics for many years [1, 2, 9]. This mainly involves the influence of the environment (thermal bath of harmonic oscillators) on the tunneling of a macroscopic particle with variable say q out of an external potential $V(q)$ which is assumed to have a metastable minimum. In most cases of physical interest, it is assumed that q interacts linearly with the environmental coordinate say x_α $(\alpha = 1, 2, \cdots)$ at a certain temperature T. The breakthrough in this subject was made by Caldeira and Leggett [2]. They considered a Euclidean Lagrangian of the form

$$\begin{aligned}\mathcal{L}_E = \frac{1}{2}M\dot{q}^2 + V(q) + \sum_\alpha \frac{1}{2}m_\alpha(\dot{x}_\alpha^2 + \omega_\alpha^2 x_\alpha^2)\\ + q\sum_\alpha c_\alpha x_\alpha,\end{aligned} \tag{1}$$

where the parameters $m_\alpha, \omega_\alpha, c_\alpha$ need not to be known in detail. The partition function is given by

$$K(q, x_\alpha; \tau) = \int \mathcal{D}q(\tau) \int \prod_\alpha \mathcal{D}x_\alpha(\tau) \exp\left(-S_E\right), \tag{2}$$

where

$$S_E = \int_0^\tau d\tau \mathcal{L}_E. \tag{3}$$

Performing the functional integral over x_α in the limit $\tau \to \infty$ gives

$$K(q; \tau) = \int \mathcal{D}q(\tau) \exp\left(-S_E^{eff}\right), \tag{4}$$

where the effective action is given by

$$\begin{aligned}S_E^{eff} &= \int_0^\tau d\tau \left[\frac{1}{2}M\dot{q}^2 + V(q)\right]\\ &+ \frac{\eta}{4\pi}\int_{-\infty}^{\infty}\int_0^\tau d\tau d\tau' \frac{[q(\tau) - q(\tau')]^2}{(\tau - \tau')^2},\end{aligned} \tag{5}$$

and η is the frictional constant. Chudnovsky [3] generalized this formalism by considering two macroscopic particles that interact with each other via a nonlinear potential $V(|x_1 - x_2|)$ with the coordinate x_2 linearly coupled to the environment. The Euclidean Lagrangian is of the form

$$\begin{aligned}\mathcal{L}_E = \frac{1}{2}M_1\dot{x}_1^2 + \frac{1}{2}M_2\dot{x}_2^2 + V(|x_1 - x_2|) + \frac{1}{2}\sum_\alpha m_\alpha \dot{x}_\alpha^2\\ + \frac{1}{2}\sum_\alpha m_\alpha\omega_\alpha^2(x_\alpha - x_2)^2.\end{aligned} \tag{6}$$

Integrating out the environmental degree of freedom and using the new coordinates

$$\begin{aligned}q &= x_1 - x_2,\\ r &= \frac{M_1x_1 + M_2x_2}{M_1 + M_2},\end{aligned} \tag{7}$$

he noticed that in the limit $M_1 \to \infty$, the effective action reduces to the form of Caldeira and Leggett Eq.(5).

In this paper, we shall generalize Chudnovsky idea by considering two coupled macroscopic particles in the plane, in the presence of a constant transverse external magnetic field and an external potential.

MODEL

We shall first consider the Euclidean Lagrangian

$$\begin{aligned}\mathcal{L}_E = \frac{m_1}{2}|\dot{\vec{x_1}}|^2 + \frac{m_2}{2}|\dot{\vec{x_2}}|^2 + i\frac{qB_\perp}{2}\left(\dot{\vec{x_1}}\times\vec{x_1} - \dot{\vec{x_2}}\times\vec{x_2}\right) \\ + \frac{1}{2}m_1\omega_1^2\vec{x_1}^{\,2} + \frac{1}{2}m_2\omega_2^2\vec{x_2}^{\,2} + V(|\vec{x_1}-\vec{x_2}|).\end{aligned} \tag{8}$$

This Lagrangian describes motion of two coupled particles in the plane, in the presence of a constant transverse external magnetic field and an external potential. These two particles interact with each other by a nonlinear potential $V(|\vec{x_1}-\vec{x_2}|)$ which has a metastable minimum. The magnetic field term in imaginary time corresponds to a symmetric gauge for the two particles. It breaks the time reversal symmetry of the Lagrangian and the weak harmonic oscillator potentials break the spatial translation symmetry of the Lagrangian, however, we can restore this symmetry (up to a total derivative) by setting $\omega_1 = \omega_2 = 0$, which will be the case at the end of our calculation, hence the total linear momentum is conserved and the dynamics of the system cannot, therefore, change the position of the center of mass [3]. Notice that the presence of the magnetic field makes the Lagrangian strictly two-dimensional.

EFFECTIVE ACTION

We proceed to the effect of an external transverse magnetic field on the tunneling of the particles out of a metastable state by following the method of Caldeira and Leggett outlined in [1]. The partition function is given by

$$Z = \int d\vec{x_1}d\vec{x_2}K(\vec{x_1},\vec{x_2},\beta), \tag{9}$$

where

$$K(\vec{x_1},\vec{x_2},\beta) = \int\mathcal{D}\vec{x_1}\int\mathcal{D}\vec{x_2}\exp\left(-S_E\right), \tag{10}$$

and

$$S_E = \int_0^\beta d\tau\mathcal{L}_E, \tag{11}$$

where $\beta = 1/T$ is the inverse temperature. The tunneling rate is proportional to $\exp(-S_E^c)$ where the Euclidean classical action S_E^c is determined by the bounce solution of the equation $\delta S_E = 0$ in which the periodic boundary condition $\vec{x_1}(0) = \vec{x_1}(\beta)$ and $\vec{x_2}(0) = \vec{x_2}(\beta)$ are required. We shall set $\hbar = 1$ throughout our calculation in this paper. Let's simplify the problem by taking $m_1 = m_2 = m$ and $\omega_1 = \omega_2 = \omega'$ and now introduce the following change of variables

$$\begin{aligned}\vec{q} &= \vec{x_1} - \vec{x_2}, \\ \vec{r} &= \frac{\vec{x_1}+\vec{x_2}}{2},\end{aligned} \tag{12}$$

where $\vec{q}$ is the position of particle 1 relative to particle 2 and $\vec{r}$ is the position vector of the center of mass of particles 1 and 2. In the new coordinate system, the Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_E = \frac{1}{2}\tilde{m}\dot{q_i}^2 + V(|q_i|) + \frac{1}{2}\tilde{m}\omega'^2 q_i^2 \\ + \frac{M}{2}\dot{r_i}^2 + \frac{M}{2}\omega'^2 r_i^2 + iqB_\perp\epsilon_{ij}\dot{r_i}q_j,\end{aligned} \tag{13}$$

where $\tilde{m} = \frac{1}{2}m$ is the reduced mass and $M = 2m$ is the total mass. We have dropped a total derivative in the Lagrangian which comes from the magnetic field term, it has no contribution to the classical equations of motion. The density matrix becomes

$$K(\vec{q},\vec{r},\beta) = \int_{\vec{q}(0)=\vec{q_0}}^{\vec{q}(\beta)=\vec{q_0}}\mathcal{D}\vec{q}\int_{\vec{r}(0)=\vec{r_0}}^{\vec{r}(\beta)=\vec{r_0}}\mathcal{D}\vec{r}\exp\left(-S_E\right). \tag{14}$$

Exploiting the periodic boundary conditions on $\vec{q}$ and $\vec{r}$, one can expand them in terms of Fourier series [4]:

$$\begin{aligned}\vec{q}(\tau) &= \frac{1}{\beta}\sum_{n=-\infty}^{n=\infty}\vec{q}_n e^{i\omega_n\tau}, \\ \vec{r}(\tau) &= \frac{1}{\beta}\sum_{n=-\infty}^{n=\infty}\vec{r}_n e^{i\omega_n\tau},\end{aligned} \tag{15}$$

where $\vec{q}_{-n} = \vec{q}_n^{\,*}$, $\vec{r}_{-n} = \vec{r}_n^{\,*}$ and $\omega_n = -\omega_{-n} = 2\pi n/\beta$ is the bosonic matsubara frequency. The classical equation of motion for $\vec{r}$ is

$$M\ddot{\bar{r}}_i + iqB_\perp\epsilon_{ij}\dot{\bar{q}}_j - M\omega'^2\bar{r}_i = 0. \tag{16}$$

Fourier transforming (16) we obtain

$$\bar{r}_{in} = -\frac{qB_\perp\omega_n\epsilon_{ij}q_{jn}}{M(\omega_n^2+\omega'^2)}. \tag{17}$$

For any path, we write

$$r_i(\tau) = \bar{r}_i(\tau) + y_i(\tau), \tag{18}$$

so $y_i(0) = y_i(\beta) = 0$. The integral over $r_i(\tau)$ becomes

$$\begin{aligned}\mathcal{S}_E^r = \int_0^\beta d\tau\left\{\frac{M}{2}\dot{\bar{r}}_i^2 + \frac{1}{2}M\omega'^2\bar{r}_i^2 + iqB_\perp\epsilon_{ij}\dot{\bar{r}}_i\bar{q}_j\right\} \\ + \int_0^\beta d\tau\left\{\frac{M}{2}\dot{y}_i^2 + \frac{1}{2}M\omega'^2 y_i^2\right\}.\end{aligned} \tag{19}$$

The linear terms in y_i vanishes by virtue of the classical equation of motion (16). Fourier transforming (19) and using (17) we obtain

$$\begin{aligned}\mathcal{S}_E^r = \frac{1}{\beta}\sum_{n=-\infty}^{n=\infty}\frac{1}{2}M(\omega_n^2+\omega'^2)|y_{in}|^2 \\ + \frac{1}{\beta}\sum_{n=-\infty}^{n=\infty}\frac{(qB_\perp\omega_n)^2}{2M(\omega_n^2+\omega'^2)}|q_{in}|^2,\end{aligned} \tag{20}$$

using the fact that $\mathcal{D}r_i = \mathcal{D}r_{in} = \mathcal{D}y_{in}$, the Gaussian integral over y_{in} is easily done, we finally obtain an effective action given by

$$\begin{aligned} S_E^{eff} &= \int_0^\beta d\tau \left\{ \frac{\tilde{m}}{2}\dot{q_i}^2 + V(|q_i|) \right\} \\ &- \frac{1}{2\beta} \sum_{n=-\infty}^{n=\infty} \mathcal{A}_n |q_{in}|^2, \end{aligned} \tag{21}$$

where

$$\mathcal{A}_n = -\left(\tilde{m}\omega'^2 + \frac{(qB_\perp \omega_n)^2}{M(\omega_n^2 + \omega'^2)} \right). \tag{22}$$

The effective action can be written equivalently as

$$\begin{aligned} S_E^{eff} &= \int_0^\beta d\tau \left\{ \frac{\tilde{m}}{2}\dot{q_i}^2 + V(|q_i|) \right\} \\ &- \frac{1}{2} \int_0^\beta d\tau \int_0^\beta d\tau' \mathcal{A}(\tau - \tau') q_i(\tau) q_i(\tau'), \end{aligned} \tag{23}$$

where

$$\mathcal{A}(\tau - \tau') = \frac{1}{\beta} \sum_{n=-\infty}^{n=\infty} \mathcal{A}_n e^{i\omega_n(\tau - \tau')}. \tag{24}$$

The first terms in (22) is independent of ω_n and thus give an unnecessary delta function contribution to (24) which does not have any contribution to the effective action and hence can be neglected. This allows one to write (24) as

$$\mathcal{A}(\tau - \tau') = -M \left(\frac{qB_\perp}{M} \right)^2 \frac{1}{\beta} \sum_{n=-\infty}^{n=\infty} \frac{\omega_n^2 e^{i\omega_n(\tau-\tau')}}{(\omega_n^2 + \omega'^2)}. \tag{25}$$

Using the fact that

$$\int_0^\beta d\tau \mathcal{A}(\tau) = 0, \tag{26}$$

the equivalent form of the Caldeira Leggett effective action [1] one can obtain from (23) is

$$\begin{aligned} S_E^{eff} &= \int_0^\beta d\tau \left\{ \frac{\tilde{m}}{2}\dot{q_i}^2 + V(|q_i|) \right\} \\ &+ \frac{1}{4} \int_0^\beta d\tau \int_0^\beta d\tau' \mathcal{A}(\tau - \tau') \left[q_i(\tau) - q_i(\tau') \right]^2. \end{aligned} \tag{27}$$

Further simplification of (25) yields

$$\begin{aligned} \mathcal{A}(\tilde{\tau}) &= M \left(\frac{qB_\perp}{M} \right)^2 \frac{1}{\beta} \sum_{n=-\infty}^{n=\infty} \left[\frac{\omega'^2}{(\omega_n^2 + \omega'^2)} - 1 \right] e^{i\omega_n \tilde{\tau}} \\ &\approx M \left(\frac{qB_\perp}{M} \right)^2 \frac{1}{\beta} \sum_{n=-\infty}^{n=\infty} \frac{\omega'^2}{(\omega_n^2 + \omega'^2)} e^{i\omega_n \tilde{\tau}}. \end{aligned} \tag{28}$$

This allows one to use the summation formula [5],

$$\mathcal{A}(\tilde{\tau}) = \frac{M\omega'}{2} \left(\frac{qB_\perp}{M} \right)^2 \frac{\cosh\left[\omega'(\beta/2 - |\tilde{\tau}|) \right]}{\sinh(\beta\omega'/2)}. \tag{29}$$

Exponentiating the hyperbolic functions we have

$$\begin{aligned} \mathcal{A}(\tilde{\tau}) = &\frac{M\omega'}{2} \left(\frac{qB_\perp}{M} \right)^2 \Big[(1 + n(\omega')) e^{-\omega'|\tilde{\tau}|} \\ &+ n(\omega') e^{\omega'|\tilde{\tau}|} \Big], \end{aligned} \tag{30}$$

where $\tilde{\tau} = \tau - \tau'$ and $n(\omega)$ is a single-particle bose distribution function given by

$$n(\omega') = \frac{1}{e^{\beta\omega'} - 1}. \tag{31}$$

We follow the method in [1] and periodically continue the path $q_i(\tau)$ outside the range $0 \leq \tau < \beta$ by the prescription $q_i(\tau + \beta) = q_i(\tau)$, then (30) becomes

$$\mathcal{A}_0(\tilde{\tau}) = \tfrac{M\omega'}{2} \left(\tfrac{qB_\perp}{M} \right)^2 e^{-\omega'|\tilde{\tau}|}, \tag{32}$$

which is the zero temperature limit of (30). We can further simplify (30) by taking the limit $\omega' \longrightarrow 0$, this gives

$$\tilde{\mathcal{A}}_0 = \tfrac{M}{\beta} \left(\tfrac{qB_\perp}{M} \right)^2. \tag{33}$$

The effective action becomes

$$\begin{aligned} S_E^{eff} &= \int_0^\beta d\tau \left\{ \frac{\tilde{m}}{2}\dot{q_i}^2 + V(|q_i|) \right\} \\ &+ \frac{M}{4\beta} \left(\frac{qB_\perp}{M} \right)^2 \int_0^\beta d\tau \int_0^\beta d\tau' \left[q_i(\tau) - q_i(\tau') \right]^2. \end{aligned} \tag{34}$$

It was shown by Ping [6] for one particle in three dimensions with a metastable potential only in the z-direction and a weak harmonic oscillator potential in x and y directions under the influence of an external magnetic field $\mathbf{B} = (B_\parallel, 0, B_\perp)$ whose vector potential is given by $\mathbf{A} = (0, B_\perp x - B_\parallel z, 0)$ that the effective action maps onto a one dimensional version of (27) and for $B_\parallel = 0$ which corresponds to Landau gauge, there is no dependence of effective action (tunneling rate) on $B_\perp$. However, for the case of two charged coupled particles with equal and opposite charges in the plane in the presence of an external potential and a traverse external magnetic field determined by a symmetric gauge, and a metastable potential given by the interaction potential, the effective action is strictly two dimensional ($i = 1, 2$) unlike the one dimensional version obtained in [1] and [3] with the inclusion of an environmental coupling in the absence of an external magnetic field.

LEGGETT'S PRESCRIPTION AT ZERO TEMPERATURE

In order to obtain the effective action at zero temperature ($\beta \to \infty$), we apply the Leggett prescription [9], which tell us that if the Fourier transform of the real time classical equation of motion is of the form

$$K(\omega)q(\omega) = -\left(\frac{dV}{dq}\right)(\omega), \tag{35}$$

then the formula for the tunneling rate can be obtained from the effective action

$$S_E^{eff} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \frac{1}{2} K(-i|\omega|)|q(\omega)|^2 + S_v(q(\omega)), \tag{36}$$

where

$$S_v(q(\omega)) \equiv \int_{-\infty}^{\infty} d\tau V(q(\tau)). \tag{37}$$

At zero temperature, the Fourier series in real time takes the form

$$\begin{aligned} \vec{q}(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, \vec{q}(\omega)e^{i\omega t}, \\ \vec{r}(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, \vec{r}(\omega)e^{i\omega t}, \end{aligned} \tag{38}$$

Now the real time equations of motion from (13) are

$$M(\ddot{r}_i + \omega'^2 r_i) - qB_\perp \epsilon_{ij}\dot{q}_j = 0. \tag{39}$$

$$\tilde{m}(\ddot{q}_i + \omega'^2 q_i) - qB_\perp \epsilon_{ij}\dot{r}_j = -\frac{dV}{dq_i}. \tag{40}$$

Next, we Fourier transform (39) and (40) and solve for $q_i(\omega)$. The result is

$$K(\omega)q_i(\omega) = -\left(\frac{dV}{dq_i}\right)(\omega), \tag{41}$$

where

$$K(\omega) = -\tilde{m}\omega^2 + \tilde{m}\omega'^2 + \frac{(qB_\perp\omega)^2}{M(\omega^2 - \omega'^2)}. \tag{42}$$

Substituting (42) into (36), we obtain

$$\begin{aligned} S_E^{eff} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \frac{1}{2}\Big[\tilde{m}(\omega^2 + \omega'^2) \\ + \frac{(qB_\perp\omega)^2}{M(\omega^2 + \omega'^2)}\Big] |q_i(\omega)|^2 + S_v(q_i(\omega)). \end{aligned} \tag{43}$$

In other to see how this result follows from (21) and (22) as $\beta \to \infty$, we first Fourier transform the kinetic term of (21) and replace ω_n by ω and the summation over n by integration over ω with a normalization factor of $1/2\pi$. Therefore, we see that the results are consistent.

DISSIPATIVE ENVIRONMENT

In this section we shall consider the coupling of the Lagrangian in (8) to a thermal harmonic oscillators in two dimensions, therefore its classical dynamics will be dissipative. The Euclidean Lagrangian we shall consider is given by

$$\begin{aligned} \mathcal{L}_E = \frac{m_1}{2}|\dot{\vec{x}}_1|^2 + \frac{m_2}{2}|\dot{\vec{x}}_2|^2 + i\frac{qB_\perp}{2}\left(\dot{\vec{x}_1}\times\vec{x}_1 - \dot{\vec{x}_2}\times\vec{x}_2\right) \\ + \frac{1}{2}m_1\omega_1^2\vec{x}_1^2 + \frac{1}{2}m_2\omega_2^2\vec{x}_2^2 + V(|\vec{x}_1 - \vec{x}_2|) \\ + \sum_{\alpha=1}^{N}\frac{m_\alpha}{2}\left[\dot{\vec{x}}_\alpha^{\,2} + \omega_\alpha^2\left(\vec{x}_\alpha - \vec{x}\right)^2\right]. \end{aligned} \tag{44}$$

where $\vec{x} = (\vec{x}_1, \vec{x}_2)$.

In the absence of a transverse magnetic field and weak harmonic oscillator potentials, the Lagrangian corresponds to a two dimensional version of the one considered in [3] except for the coupling in $\vec{x}$ instead of $\vec{x}_2$. The problem of one particle in three dimensions with a metastable potential only in the z-direction and a weak harmonic oscillator potential in x and y directions under the influence of an external magnetic field $\mathbf{B} = (B_\parallel, 0, B_\perp)$ whose vector potential (Landua gauge) is given by $\mathbf{A} = (0, B_\perp x - B_\parallel z, 0)$, coupled to an environmental harmonic oscillators has been considered in [7]. It was shown that the problem can be mapped onto a one dimensional one. In (44) we have assumed that only particle 2 is coupled to a large environmental harmonic oscillators. Notice that the Lagrangian is still translational invariant up to a total derivative when $\omega_1 = \omega_2 = 0$. In order to obtain the effective action, we expand $\vec{x}_\alpha$ and $\vec{x}$ in a Fourier series as

$$\begin{aligned} \vec{x}_\alpha(\tau) &= \frac{1}{\beta}\sum_{n=-\infty}^{n=\infty}\vec{x}_{\alpha n}e^{i\omega_n\tau} \\ \vec{x}(\tau) &= \frac{1}{\beta}\sum_{n=-\infty}^{n=\infty}\vec{x}_n e^{i\omega_n\tau}, \quad \text{etc} \end{aligned} \tag{45}$$

and perform the Gaussian integration over $\vec{x}_{\alpha n}$. This gives

$$K(\vec{x_1}, \vec{x_2}, \beta) = \int \mathcal{D}\vec{x_1}(\tau)\int \mathcal{D}\vec{x_2}(\tau)\exp\left(-S_E\right), \tag{46}$$

where

$$\begin{aligned} S_E = \int_0^\beta d\tau\, \Big[\frac{m}{2}(\dot{\vec{x}}_1^{\,2} + \dot{\vec{x}}_2^{\,2}) + i\frac{qB_\perp}{2}\left(\dot{\vec{x_1}}\times\vec{x_1} - \dot{\vec{x_2}}\times\vec{x_2}\right) \\ + \frac{1}{2}(m\omega'^2 + \sum_\alpha m_\alpha\omega_\alpha^2)\vec{x}^2 + V(|\vec{x_1} - \vec{x_2}|)\Big] \\ - \frac{1}{\beta}\sum_{\alpha n}\frac{m_\alpha\omega_\alpha^4}{2(\omega_n^2 + \omega_\alpha^2)}|\vec{x}_n|^2. \end{aligned} \tag{47}$$

We have set $m_1 = m_2 = m$ and $\omega_1 = \omega_2 = \omega'$ to arrive at this result. Next, we rewrite (47) in terms of $\vec{q}$ and $\vec{r}$ using (12), Fourier transform using (15) and use the fact that the Fourier coefficient $|\vec{x}_n|^2 = |\vec{x}_{1n}|^2 + |\vec{x}_{2n}|^2$ and

$$\vec{x}_{1n} = \vec{r}_n + \frac{1}{2}\vec{q}_n, \tag{48}$$

$$\vec{x}_{2n} = \vec{r}_n - \frac{1}{2}\vec{q}_n. \tag{49}$$

Then the Gaussian integration over $\vec{r}_n$ can be easily done, and we obtain

$$K(\vec{q},\vec{r};\beta) = \int \mathcal{D}\vec{q}(\tau)\ \exp\left(-S_E^{eff}\right), \tag{50}$$

where

$$\begin{aligned} S_E^{eff} &= \int_0^\beta d\tau \left(\frac{\tilde{m}}{2}\dot{\vec{q}}^2 + V(|\vec{q}|)\right) \\ &+ \frac{1}{2}\int_0^\beta d\tau \int_0^\beta d\tau' \mathcal{A}(\tau-\tau')\vec{q}(\tau)\vec{q}(\tau') \\ &+ \frac{1}{2}\int_0^\beta d\tau \int_0^\beta d\tau' \mathcal{B}(\tau-\tau')\dot{\vec{q}}(\tau)\dot{\vec{q}}(\tau'), \end{aligned} \tag{51}$$

and

$$\mathcal{A}(\tau-\tau') = \frac{1}{\beta}\sum_n \mathcal{A}_n e^{i\omega_n(\tau-\tau')} \tag{52}$$

$$\mathcal{B}(\tau-\tau') = \frac{1}{\beta}\sum_n \mathcal{B}_n e^{i\omega_n(\tau-\tau')}. \tag{53}$$

In this case the $\mathcal{A}_n$ and $\mathcal{B}_n$ are given by

$$\mathcal{A}_n = \frac{(qB_\perp\omega_n)^2}{M(\omega_n^2+\omega'^2) + 2\lambda_n(\beta)\omega_n^2}, \tag{54}$$

$$\mathcal{B}_n = \frac{1}{2}\lambda_n(\beta). \tag{55}$$

and

$$\lambda_n(\beta) = \sum_\alpha \frac{m_\alpha\omega_\alpha^2}{\omega_\alpha^2+\omega_n^2}. \tag{56}$$

We have dropped a constant term independent of ω_n since it have no contribution to (51).

Now let's consider the limit of weak dissipation, that is $\lambda_n(\beta) << M(\omega_n^2+\omega'^2)$, we have

$$\frac{(qB_\perp\omega_n)^2}{M(\omega_n^2+\omega'^2)+2\omega_n^2\lambda_n(\beta)} \approx \frac{(qB_\perp\omega_n)^2}{M(\omega_n^2+\omega'^2)}. \tag{57}$$

We then substitute (57) into (54) and use the definition of $\lambda_n(\beta)$ and the summation formula [4]

$$\frac{1}{\beta}\sum_n \frac{m_\alpha\omega_\alpha^2}{\omega_\alpha^2+\omega_n^2}e^{i\omega_n\tilde{\tau}} = \frac{m_\alpha\omega_\alpha}{2}\frac{\cosh\left[\omega_\alpha(\beta/2-\tilde{\tau})\right]}{\sinh(\omega_\alpha\beta/2)}, \tag{58}$$

we obtain in the limit $\omega' \to 0$:

$$\begin{aligned} S_E^{eff} &= \int_0^\beta d\tau \left(\frac{\tilde{m}}{2}\dot{\vec{q}}^2 + V(|\vec{q}|)\right) \\ &+ \frac{M\omega_c^2}{4\beta}\int_{-\infty}^{\infty} d\tau' \int_0^\beta d\tau \left[\vec{q}(\tau)-\vec{q}(\tau')\right]^2 \\ &+ \frac{1}{4}\int_{-\infty}^{\infty} d\tau' \int_0^\beta d\tau \alpha(\tau-\tau')\left[\vec{q}(\tau)-\vec{q}(\tau')\right]^2 \end{aligned} \tag{59}$$

where

$$\alpha(\tau-\tau') = \frac{1}{4}\sum_\alpha m_\alpha\omega_\alpha^3 \exp(-\omega_\alpha|\tau-\tau'|) \tag{60}$$

and $\omega_c = qB_\perp/M$ is the cyclotron frequency.

OHMIC DISSIPATION

In line with [3], we shall assume that the effect of the oscillators on the motion of the particles result in the force of friction $\eta\dot{\vec{x}}$. This requires that the spectral density should be defined as

$$J(\tilde{\omega}) = \frac{\pi}{2}\sum_\alpha m_\alpha\omega_\alpha^3\delta(\tilde{\omega}-\omega_\alpha). \tag{61}$$

All the information concerning the effect of the environment on the dynamics of the particles is contained in $J(\tilde{\omega})$. We shall consider the case of ohmic dissipation in which $J(\tilde{\omega})$ is given by

$$J(\tilde{\omega}) = \eta\tilde{\omega}, \tag{62}$$

up to a frequency cutoff ω_c. Using (61) we have

$$\lambda_n(\beta) = \frac{2}{\pi}\int_0^\infty \frac{d\tilde{\omega}}{\tilde{\omega}}\frac{J(\tilde{\omega})}{\tilde{\omega}^2+\omega_n^2}. \tag{63}$$

Substituting (62) into (63) and performing the integration, we obtain

$$\lambda_n = \eta/|\omega_n|. \tag{64}$$

Substituting this expression into the expressions for $\mathcal{A}_n$ and $\mathcal{B}_n$ and setting $\omega' = 0$ we have

$$\begin{aligned} S_E^{eff} &= \int_0^\beta d\tau \left(\frac{\tilde{m}}{2}\dot{\vec{q}}^2 + V(|\vec{q}|)\right) \\ &+ \frac{1}{4}\int_0^\beta d\tau \int_0^\beta d\tau' \mathcal{A}(\tau-\tau')\left[\vec{q}(\tau)-\vec{q}(\tau')\right]^2 \\ &+ \frac{1}{4}\int_0^\beta d\tau \int_0^\beta d\tau' \mathcal{B}(\tau-\tau')\left[\vec{q}(\tau)-\vec{q}(\tau')\right]^2, \end{aligned} \tag{65}$$

where

$$\mathcal{A}(\tau-\tau') = -\frac{1}{\beta}\sum_n \frac{(qB_\perp\omega_n)^2}{M\omega_n^2+2\eta|\omega_n|}e^{i\omega_n(\tau-\tau')}, \tag{66}$$

$$\mathcal{B}(\tau-\tau') = -\frac{1}{\beta}\sum_n \frac{\eta|\omega_n|}{2}e^{i\omega_n(\tau-\tau')}. \tag{67}$$

Let us now work out the effective action at $T = 0$ for the case in which the motion of the two identical particles are damped by Ohmic friction with constant η respectively. We shall do this by applying the Leggett's prescription outline above. The real time classical equations of motion are

$$m\ddot{x}_i^1 - qB_\perp \epsilon_{ij}\dot{x}_j^1 + \eta\dot{x}_i^1 = -\frac{\partial V}{\partial x_i^1}, \tag{68}$$

$$m\ddot{x}_i^2 + qB_\perp \epsilon_{ij}\dot{x}_j^2 + \eta\dot{x}_i^2 = -\frac{\partial V}{\partial x_i^2}. \tag{69}$$

Substituting x_i^1 and x_i^2 in terms of r_i and q_i using the inverse transform of (12), we obtain, after adding and subtracting the resulting equations

$$\begin{aligned} &2m\ddot{r}_i - qB_\perp \epsilon_{ij}\dot{q}_j + 2\eta\dot{r}_i = 0, \\ &\frac{1}{2}m\ddot{q}_i - qB_\perp \epsilon_{ij}\dot{r}_j + \frac{1}{2}\eta\dot{q}_i = -\frac{\partial V}{\partial q_i}. \end{aligned} \tag{70}$$

Fourier transforming (70) and solving for $q_i(\omega)$ we obtain

$$K(\omega)q_i(\omega) = -\frac{\partial V}{\partial q_i}(\omega), \tag{71}$$

where $K(\omega)$ in this case is given by

$$K(\omega) = -\tilde{m}\omega^2 + \frac{(qB_\perp\omega)^2}{-M\omega^2 + 2i\eta\omega} + i\eta\omega. \tag{72}$$

The effective action at $T = 0$ is then given by

$$\begin{aligned} S_E^{eff} = &\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \frac{1}{2}\left[\tilde{m}\omega^2 + \frac{(qB_\perp\omega)^2}{M\omega^2 + 2\eta|\omega|}\right. \\ &\left. + \frac{\eta|\omega|}{2}\right]|q_i(\omega)|^2 + S_v(q_i(\omega)). \end{aligned} \tag{73}$$

We also noticed that the effective action found in [10] for the case of no pinning and finite dissipation at $T = 0$ does not give the same expression using Leggett's prescription at $T = 0$ for the case of Ohmic dissipation.

Now consider the limit $B_\perp = 0$, in this limit, the explicit expression of (67) can be easily obtained. The effective action (65) becomes

$$\begin{aligned} S_E^{eff} = &\int_0^\beta d\tau\left(\frac{1}{2}\tilde{m}\dot{\vec{q}}^2 + V(|\vec{q}|)\right) \\ &+ \frac{\eta}{8\pi}\int_0^\beta d\tau'\int_0^\beta d\tau \frac{[\vec{q}(\tau) - \vec{q}(\tau')]^2}{(\beta/\pi)^2\sin^2(\pi\tilde{\tau}/\beta)} \end{aligned} \tag{74}$$

This effective action is similar to a two dimensional version of the $m_1 = m_2$ limit of Eq.(44) in Ref.[3]. Now, consider the limit of strong dissipation $M\omega_n^2 << 2\eta|\omega_n|$, in this limit, the explicit expressions in (66) and (67) can be summed. The effective action becomes

$$\begin{aligned} S_E^{eff} = &\int_0^\beta d\tau\left(\frac{1}{2}\tilde{m}\dot{\vec{q}}^2 + V(|\vec{q}|)\right) \\ &+ \frac{\eta_{eff}}{8\pi}\int_0^\beta d\tau'\int_0^\beta d\tau \frac{[\vec{q}(\tau) - \vec{q}(\tau')]^2}{(\beta/\pi)^2\sin^2(\pi\tilde{\tau}/\beta)}, \end{aligned} \tag{75}$$

where

$$\eta_{eff} = \frac{(qB_\perp)^2}{\eta} + \eta. \tag{76}$$

CONCLUSIONS

We have studied the macroscopic quantum tunneling of two coupled particles of identical masses but opposite charges in the plane, in the presence of a constant transverse external magnetic field and an external potential whose interaction potential allows for a metastable state. We showed that the effect of the magnetic is to suppress the tunneling of the particles out of a metastable state and also our effective action remains two dimensional unlike a one dimension version obtained in [6]. We further coupled the system to a thermal bath of harmonic oscillator and showed that in the limit of weak dissipation, there is an effect of the magnetic field to the effective action. In the limit of zero magnetic field, we reproduced a two dimensional version of the results obtained in [3] for $m_1 = m_1$, which also coincides with the results of Caldeira and Leggett [1] in two dimensions in terms of the reduced mass. Our results are as a consequence of the conservation of total linear momentum as was first shown in [3] since the magnetic field term is translational invariant up to a total derivative. The results obtained here can also be applied to the metastable states of the molecules of fluid and solids in a transverse magnetic field and coupled superconductor vortex tunneling in two dimensions.

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